Transmit Power Optimization for Two-Way Relay Channels With Physical-Layer Network Coding

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***Abstract*—In this letter, we consider a two-way relay channel where two source nodes exchange their packets via a half-duplex relay node, which adopts physical-layer network coding (PNC) for exchanging packets in two time slots. Convolutional codes (CCs) are assumed to be applied as a channel code for each packet. The relay node directly decodes the XORed version of packets of two source nodes in the multiple access (MA) phase. We first mathematically analyze a bit error rate (BER) of the MA phase in the PNC with CCs in Rayleigh fading channels. Then, we propose a power allocation (PA) strategy for minimizing the derived BER expression at the relay node. It is shown that the proposed transmit power solution satisfies the following relationship: $\frac{P_1^*}{P_2^*} = \sqrt{\frac{\Omega_2}{\Omega_1}}$, where P_i^* and Ω_i denote the optimal power of the ith source node and the variance of the channel gains between the ith source node and the relay node. The proposed PA strategy significantly outperforms conventional PA schemes in terms of the BER.**

***Index Terms*—Two-way relay channel, physical-layer network coding, convolutional codes, BER, optimal power allocation.**

I. Introduction

PHYSICAL-LAYER network coding (PNC) has received much attention since it significantly increases spectral efficiency of a two-way relay network (TWRN) [1], [2]. The authors of [1] assumed that the wireless channel is an additive white Gaussian noise (AWGN) channel by exploiting *pre-equalization* procedure at sources. However, it may not be feasible for practical wireless communication systems in fast and/or frequency-selective fading environments because the source nodes may not have a channel state information (CSI) before transmission. Koike-Akino *et al.* proposed an optimized constellation design for PNC in fading channels without pre-equalization [3] and extended their work to the system with convolutional codes in [4]. In these schemes, however, the sources need to know ratios of instantaneous channel gain amplitudes of two links before transmission, which are impossible to be obtained in fast fading channels or result in heavy feedback overhead in frequency-selective fading channels. Therefore, practical PNC techniques not requesting CSI at the sources are of interest.

A practical PNC technique without pre-equalization was proposed for fast fading channels, but its performance was evaluated with only computer simulations [5]. Ju *et al.* [6] analyzed *uncoded* bit error rate (BER) of the PNC with in Rayleigh fading channels without pre-equalization or constellation optimization at the source nodes [6]. The BER performance in fading channels is known to be awful without exploiting channel codes. To *et al.* [7] proposed a combined architecture of convolutional codes (CCs) and the PNC, and they evaluated the BER performance through computer simulations in fading channels. The error performance of channel-coded physical-layer network coding scheme was analyzed in AWGN channels [10] and in quasi-static channels [11], [12] but not in fast-fading channels. Furthermore, transmit power optimization techniques at the source nodes have been investigated in [8], [9], but the schemes assume slow fading channels and also require full CSI at transmitters (CSIT). To the best of our knowledge, there has been no mathematical analysis of BER of PNC with channel codes and no power allocation (PA) strategy in *fast* fading channels.

In this letter, therefore, we mathematically analyze the BER of the PNC with CCs in fast fading channels. Based on the BER analysis, we also propose an PA strategy in order to minimize the derived BER under sum power constraint at the source nodes, which only requires CSI at receivers (CSIR). Note that the proposed PA technique only requires CSIR and the sources are assumed to know channel statistics (variance).

II. System Model

We consider a TWRN consisting of two source nodes and a relay node. All nodes are assumed to transmit an information-bit sequence with the same size, adopt the same CC, and use a binary phase shift keying (BPSK) modulation.[1] Fig. 1 shows the overall procedure of the first phase of PNC with CCs, which is called multiple access (MA) phase. The second phase, called broadcast (BC) phase, of PNC is identical to the conventional wireless communications and we focus on the MA phase in Fig. 1. N_1 and N_2 denote the source nodes, respectively, and N_R denotes the relay node. $\mathbf{u}_i$, $\mathbf{v}_i$ and $\mathbf{m}_i$ indicate information-bit sequence, codeword, and modulated symbol vector of N_i, respectively. $V(\cdot)$ and $M(\cdot)$ denote the encoding function and the modulation function, respectively. Then, we obtain that $\mathbf{v}_i = V(\mathbf{u}_i)$ and $\mathbf{m}_i = M(\mathbf{v}_i)$. We assume a BPSK mapping rule that $m_{i,n} = 1(-1)$ for $v_{i,n} = 0(1)$ respectively, where $m_{i,n}$ and $v_{i,n}$ indicate the n-th elements of $\mathbf{m}_i$ and $\mathbf{v}_i$, respectively. After the BPSK modulation, two source nodes simultaneously transmit $\mathbf{m}_1$ and $\mathbf{m}_2$ and N_R receives $\mathbf{y}_\mathrm{R}$ which is the superposition of $\mathbf{m}_1$ and $\mathbf{m}_2$ through fading channels. Let $h_{i,n}$ denote

[1]The PNC studies in [6], [7], [9]–[11] also considered BPSK.

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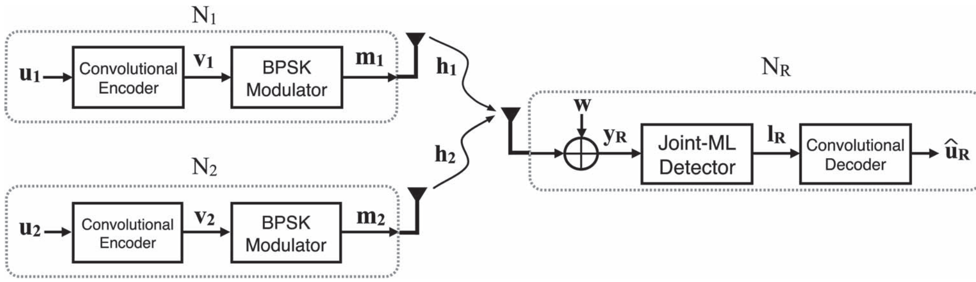

Fig. 1. Transmission/reception procedure of multiple access phase in PNC with CCs.

the channel gain between N_i and N_R at the n-th symbol. We assume that the channel severely fluctuates in a codeword (*fast fading*) and a channel interleaver is used. Then, we can model the n-th channel gain of the i-th source as an independent, zero-mean, complex Gaussian random variable with variance Ω_i, i.e. $h_{i,n} \sim \mathcal{CN}(0,\Omega_i)\ \forall n$. Then, the n-th received symbol at N_R is expressed as:

$$y_{\mathrm{R},n} = h_{1,n}\sqrt{P_1}m_{1,n} + h_{2,n}\sqrt{P_2}m_{2,n} + w_n, \tag{1}$$

where P_i denotes the transmit power of N_i and w_n represents AWGN at the n-th symbol, i.e. $w_n \sim \mathcal{CN}(0,\sigma_w^2)$. We assume the relay node exactly knows CSIs. Considering bit-wise exclusive OR (XOR, $\oplus$) as a PNC operation, the final goal of N_R is to obtain $\mathbf{u}_\mathrm{R} = \mathbf{u}_1 \oplus \mathbf{u}_2$. By using the linearity of CCs, we obtain the following relationships: $\mathbf{v}_\mathrm{R} = V(\mathbf{u}_\mathrm{R}) = V(\mathbf{u}_1 \oplus \mathbf{u}_2) = V(\mathbf{u}_1) \oplus V(\mathbf{u}_2) = \mathbf{v}_1 \oplus \mathbf{v}_2$. Then, the LLR value for $v_{\mathrm{R},n}$ is given as

$$\begin{aligned} l_{\mathrm{R},n} &= \log \frac{\mathbf{Pr}(v_{\mathrm{R},n}=0|y_{\mathrm{R},n})}{\mathbf{Pr}(v_{\mathrm{R},n}=1|y_{\mathrm{R},n})} \\ &= \log \frac{\mathbf{Pr}(v_{1,n}=0, v_{2,n}=0|y_{\mathrm{R},n}) + \mathbf{Pr}(v_{1,n}=1, v_{2,n}=1|y_{\mathrm{R},n})}{\mathbf{Pr}(v_{1,n}=1, v_{2,n}=0|y_{\mathrm{R},n}) + \mathbf{Pr}(v_{1,n}=0, v_{2,n}=1|y_{\mathrm{R},n})} \\ &= \log \frac{\exp\left(-\frac{|y_{\mathrm{R},n}-C_{\{1,1\}}|^2}{\sigma_w^2}\right) + \exp\left(-\frac{|y_{\mathrm{R},n}-C_{\{-1,-1\}}|^2}{\sigma_w^2}\right)}{\exp\left(-\frac{|y_{\mathrm{R},n}-C_{\{-1,1\}}|^2}{\sigma_w^2}\right) + \exp\left(-\frac{|y_{\mathrm{R},n}-C_{\{1,-1\}}|^2}{\sigma_w^2}\right)}, \end{aligned} \tag{2}$$

where $C_{\{m_1,m_2\}}$ is the constellation point which is expressed as

$$C_{\{m_1,m_2\}} = h_{1,n}\sqrt{P_1}m_1 + h_{2,n}\sqrt{P_2}m_2, m_i \in \{1,-1\}. \tag{3}$$

$\mathbf{l}_\mathrm{R}$ is inserted into the soft Viterbi decoder, and then N_R obtains $\hat{\mathbf{u}}_\mathrm{R}$.

In the BC phase, N_R encodes $\mathbf{u}_\mathrm{R}$ into $\mathbf{v}_\mathrm{R}$ and converts $\mathbf{v}_\mathrm{R}$ into $\mathbf{m}_\mathrm{R}$. N_R broadcasts $\mathbf{m}_\mathrm{R}$ to both source nodes. Each source node decodes the received symbol sequence and obtains $\mathbf{u}_\mathrm{R}$ through the LLR calculation and the Viterbi decoder. N_1 obtains $\mathbf{u}_2$ through $\mathbf{u}_\mathrm{R} \oplus \mathbf{u}_1$. Similarly, N_2 can obtain $\mathbf{u}_1$.

III. BER Analysis of PNC With CCs

For the BC phase, the BER performance is analyzed by using the same methodology in [13]. Hence, we focus on the performance of the MA phase in this letter.

Theorem 1: The BER performance of PNC with CCs in the MA phase is approximated at high signal-to-noise ratio (SNR) as:

$$\mathcal{P}_\mathrm{b}^{\mathrm{A1}} \approx \sum_{d_\mathrm{H}=d_\mathrm{f}}^{\infty} B_{d_\mathrm{H}} \left(\frac{1-\mu_1}{2}\right)^{d_\mathrm{H}} \sum_{k=0}^{d_\mathrm{H}-1} \binom{d_\mathrm{H}-1+k}{k} \left(\frac{1+\mu_1}{2}\right)^k,$$

where $\mu_1 = \sqrt{\frac{1}{\alpha_1+1}}$, $\alpha_1 = \frac{\sigma_w^2}{\Omega_1 P_1} + \frac{\sigma_w^2}{\Omega_2 P_2}$. Furthermore, B_{d_H} and d_f represent the total number of non-zero information bits on all weight-d_H codewords and the minimum Hamming distance (free distance) between any two different codewords which are included in the codeword set, respectively. We call this Approx-1 BER.

Proof: The BER performance of CCs was expressed as [14]

$$\mathcal{P}_\mathrm{b} \le \sum_{d_\mathrm{H}=d_\mathrm{f}}^{\infty} B_{d_\mathrm{H}} \mathcal{P}_\mathrm{pair}(d_\mathrm{H}), \tag{4}$$

where $\mathcal{P}_\mathrm{pair}(d_\mathrm{H})$ denotes the pair-wise error probability between two codewords whose Hamming distance is d_H, In order to approximately derive $\mathcal{P}_\mathrm{pair}(d_\mathrm{H})$, we need to know the Euclidean distance between two codewords whose Hamming distance is d_H. Without loss of generality, we assume that all-zero codewords are transmitted from the sources ($\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{0}$, then $\mathbf{v}_\mathrm{R} = \mathbf{0}$). Let assume that $\mathbf{v}'_\mathrm{R}$ has d_H non-zero bits and $< i >$-th components of $\mathbf{v}_\mathrm{R}$ and $\mathbf{v}'_\mathrm{R}$ are different from each other for $i = 1, \cdots, d_\mathrm{H}$. Since $v_{1,<i>}$ and $v_{2,<i>}$ are equal to '0', the right constellation point is $C_{\{1,1\}}$ and miss-detection events correspond to $(\widehat{C} = C_{\{1,-1\}})$ and $(\widehat{C} = C_{\{-1,1\}})$, where $\widehat{C}$ is the estimation for the right constellation point. $(\widehat{C} = C_{\{-1,-1\}})$ belongs to the wrong estimation event but is not an miss-detection event. Miss-detection events are dominantly caused by miss-detection of $C_{\{1,1\}}$ with the closest constellation point to $C_{\{1,1\}}$ especially at high SNR such that if $|C_{\{1,1\}} - C_{\{1,-1\}}| > |C_{\{1,1\}} - C_{\{-1,1\}}|$, $\mathbf{Pr}(\widehat{C} = C_{\{1,-1\}}) \ll \mathbf{Pr}(\widehat{C} = C_{\{-1,1\}})$ and vice versa. In this respect, we take into account only the closer one to $C_{\{1,1\}}$ between $C_{\{1,-1\}}$ and $C_{\{-1,1\}}$. Therefore, the Euclidean distance between $C_{\{1,1\}}$ and $\widehat{C}$ for the i-th miss-detection symbol is approximated at high SNR as

$$d_{E,<i>} \approx \min\left(\left|C_{\{1,1\}} - C_{\{1,-1\}}\right|, \left|C_{\{1,1\}} - C_{\{-1,1\}}\right|\right). \tag{5}$$

From (3), $d_{E,<i>} \approx \min(2|h_{1,<i>}\sqrt{P_1}|, 2|h_{2,<i>}\sqrt{P_2}|)$. Let $Z = \min(|h_{1,<i>}\sqrt{P_1}|, |h_{2,<i>}\sqrt{P_2}|)$, then Z becomes the minimum of two Rayleigh distributed random variables. Z is also an i.i.d. Rayleigh distributed random variable and its probability density function (PDF) can be expressed as (See Appendix)

$$f_Z(z) = 2\left(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_2 P_2}\right) z \cdot e^{-\left(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_2 P_2}\right) z^2}. \tag{6}$$

We can express the Euclidean distance of two codewords whose Hamming distance is d_H as follows:

$$d_\mathrm{E}(d_\mathrm{H}) = \sqrt{\sum_{i=1}^{d_\mathrm{H}} d_{E,<i>}^2} \approx \sqrt{4\sum_{i=1}^{d_\mathrm{H}} Z_i^2}. \tag{7}$$

We let $\mathcal{X} = \sum_{i=1}^{d_\mathrm{H}} Z_i^2$. Since Z_i^2 follows an exponential distribution with rate $(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_2 P_2})$, $\mathcal{X}$ becomes an Erlang-distributed random variable with shape $d_{\rm H}$ and rate $\lambda = (\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_2 P_2})$ whose PDF is given by $f(\mathcal{X}) = \frac{1}{(d_{\rm H}-1)!}\lambda^{d_{\rm H}}\mathcal{X}^{d_{\rm H}-1}e^{-\lambda\mathcal{X}}$ [15]. Then, we can derive the pairwise error probability of two codewords whose Euclidean distance is $d_{\rm E}(d_{\rm H})$ as $Q\left(\frac{d_{\rm E}(d_{\rm H})/2}{\sqrt{\sigma_w^2/2}}\right) \approx Q\left(\sqrt{\frac{2\mathcal{X}}{\sigma_w^2}}\right)$. Since $\mathcal{X}$ is a random variable, we average $Q\left(\sqrt{\frac{2\mathcal{X}}{\sigma_w^2}}\right)$ over $\mathcal{X}$, then $\mathcal{P}_{\rm pair}(d_{\rm H})$ is approximated as follows: (see Eq. (3.37) in [16])

$$\begin{aligned}\mathcal{P}_{\rm pair}(d_{\rm H}) &\approx E\left[Q\left(\sqrt{\frac{2\mathcal{X}}{\sigma_w^2}}\right)\right] \\ &= \left(\frac{1-\mu_1}{2}\right)^{d_{\rm H}} \sum_{k=0}^{d_{\rm H}-1} \binom{d_{\rm H}-1+k}{k} \left(\frac{1+\mu_1}{2}\right)^{k}, \quad (8)\end{aligned}$$

where $\mu_1 = \sqrt{\frac{1}{\alpha_1+1}}$, and $\alpha_1 = \frac{\sigma_w^2}{\Omega_1 P_1} + \frac{\sigma_w^2}{\Omega_2 P_2}$. P_i is equal to $R_{\rm C} P_{b,i}$, where $R_{\rm C}$ denotes the coding rate and $P_{b,i}$ denotes the allocated transmit power on an information-bit at N_i. Finally, we can obtain the approximated BER of the PNC scheme in the MA phase by substituting (8) into (4), which completes proof. ■

Using Taylor's series expansion, (8) can be approximated at high SNR as follows [17]:

$$\mathcal{P}_{\rm pair}(d_{\rm H}) \approx \binom{2d_{\rm H}-1}{d_{\rm H}} \left(\frac{\alpha_1}{4}\right)^{d_{\rm H}}. \quad (9)$$

In addition, at high SNRs, most errors are caused by miss-detection with the nearest codeword, i.e. the codeword whose Hamming distance is $d_{\rm f}$. Then, we can obtain the following remark.

Remark 1: For high SNRs, the BER of PNC with CCs at high SNR is further approximated as:

$$\mathcal{P}_{\rm b}^{\rm A2} \approx B_{d_{\rm f}} \binom{2d_{\rm f}-1}{d_{\rm f}} \left(\frac{\alpha_1}{4}\right)^{d_{\rm f}} = B_{d_{\rm f}} \binom{2d_{\rm f}-1}{d_{\rm f}} \left(\frac{1}{4\Upsilon_1} + \frac{1}{4\Upsilon_2}\right)^{d_{\rm f}}, \quad (10)$$

where $\Upsilon_i = \Omega_i P_i/\sigma_w^2 = R_{\rm C}\Omega_i P_{{\rm b},i}/\sigma_w^2$, and this is called the Approx-2 BER.

IV. Transmit Power Allocation

In this section, we investigate a transmit power optimization at the source nodes in order to minimize the BER of PNC in the MA phase for a given sum power constraint. Setting the Approx-2 BER as an objective function, the optimization problem of transmit power **[Pro-TP]** is formulated as **[Pro-TP]**:

$$\min_{(P_1,P_2>0)} f_0(P_1,P_2) = A\left(\frac{1}{4\Omega_1 P_1/\sigma_w^2} + \frac{1}{4\Omega_2 P_2/\sigma_w^2}\right)^{d_{\rm f}}$$
$$\text{subject to } f_1(P_1,P_2) = P_1 + P_2 - \widetilde{P} \le 0,$$

where A represents $B_{d_{\rm f}} \cdot \binom{2d_{\rm f}-1}{d_{\rm f}}$ and $\widetilde{P}$ denotes the constraint on the maximum total power consumed by source nodes per symbol.

Theorem 2: The optimal solution of [**Pro-TP**] is given as

$$(P_1^*, P_2^*) = \left(\frac{\sqrt{\Omega_2}}{\sqrt{\Omega_1}+\sqrt{\Omega_2}}\widetilde{P}, \frac{\sqrt{\Omega_1}}{\sqrt{\Omega_1}+\sqrt{\Omega_2}}\widetilde{P}\right). \quad (11)$$

Proof: $f_0(P_1,P_2)$ is strictly convex since its second derivative $\nabla^2 f_0$ is positive definite, and $f_1(P_1,P_2)$ is also convex.

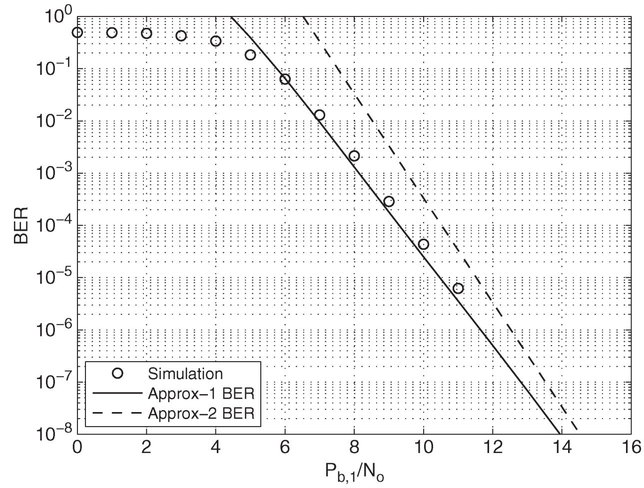

Fig. 2. Two approximated BERs of the PNC with BPSK modulation for $P_{\rm b,1} = P_{\rm b,2}$ and $\Omega_1 = \Omega_2 = 1$, with a (133, 171) convolutional code.

Therefore, we can define the Lagrangian:

$$L(\mathbf{P},\lambda) = f_0(P_1,P_2) + \lambda f_1(P_1,P_2), \quad (12)$$

where $\lambda > 0$ denotes a Lagrange multiplier. Finding a point (P_1,P_2) satisfying $\frac{dL(\mathbf{P},\lambda)}{dP_1} = \frac{dL\mathbf{P},\lambda)}{dP_2} = 0$, we obtain the optimizer:

$$P_1^* = \frac{1}{\sqrt{\Omega_1}}\left[A\left(\frac{\sigma_w^2}{4}\right)^{d_{\rm f}} d_{\rm f}\left(\frac{1}{\sqrt{\Omega_1}} + \frac{1}{\sqrt{\Omega_2}}\right)^{d_{\rm f}-1}\right]\lambda^{-\frac{1}{d_{\rm f}+1}}$$
$$P_2^* = \frac{1}{\sqrt{\Omega_2}}\left[A\left(\frac{\sigma_w^2}{4}\right)^{d_{\rm f}} d_{\rm f}\left(\frac{1}{\sqrt{\Omega_1}} + \frac{1}{\sqrt{\Omega_2}}\right)^{d_{\rm f}-1}\right]\lambda^{-\frac{1}{d_{\rm f}+1}}$$

the ratio between P_1^* and P_2^* is given by

$$\frac{P_1^*}{P_2^*} = \sqrt{\frac{\Omega_2}{\Omega_1}}. \quad (13)$$

Since the BER performance is minimized when the maximum power is used, the optimal solution needs to satisfy $P_1^* + P_2^* = \widetilde{P}$. Therefore, we easily find the optimal solution using (13). ■

Remark 2: From (13), the ratio between the optimal transmit powers at the source nodes is the inverse of the ratio between the standard deviations of channel gains between the source nodes and the relay node.

V. Numerical Examples

We utilize a convolutional code with $R_{\rm C} = 1/2$ and a generator polynomial (133, 171) in octal number, and the $d_{\rm f}$ is set to 10. The weight enumerate function (WEF) of this code is given by $36X^{10} + 211X^{12} + 1404X^{14} + 11633X^{16} + 77433X^{18} + \cdots$ in [18]. Fig. 2 shows the two approximated BERs of the PNC for $P_{\rm b,1} = P_{\rm b,2}$ and $\Omega_1 = \Omega_2 = 1$. Note that $P_{{\rm b},i} = P_i/R_{\rm C}$. The Approx-1 BER result agrees very well with the simulation result in the PNC case. The Approx-2 BER result has approximately 1 dB gap, compared with the simulation value at a BER of 10^{-5}, but the gap is reduced as the $P_{\rm b,1}/\sigma_w^2$ increases.

In order to evaluate the proposed power allocation methods for varying variances of channel gains, we model the variances as a function of distance [19]: $\Omega_i(d_i) = \left(\frac{c/f_{\rm c}}{4\pi d_0}\right)^2 \cdot \left(\frac{d_i}{d_0}\right)^{\gamma}$, where $c = 3\times10^8$ m/s, $f_{\rm c}$, d_i, d_o and γ denote the speed of light, the carrier frequency, the distance between N_i and $N_{\rm R}$, a

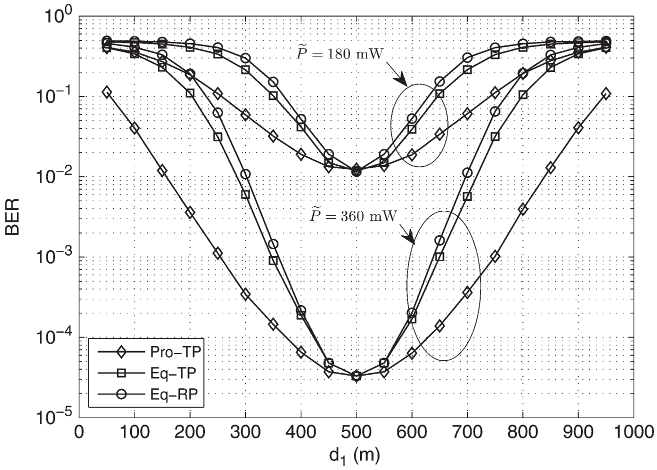

Fig. 3. BER performance of the MA phase of three transmit power allocation methods for varying d_1 for given $\widetilde{P} = 180$ and 360 mW when $d_{12} = 1000$ m.

reference distance, and the path-loss exponent, respectively. As a representative simulation example, we set $f_c = 900$ MHz, $d_0 = 10$ m, $\gamma = 4$. In addition, we use $BW = 10$ MHz and $\mathcal{N}_0 = -204$ dBW/Hz as the bandwidth and the noise spectral density. The distance between two source nodes, d_{12}, is fixed to 1000 m and the relay node moves on the straight line between those two source nodes.

For comparison, we formulate two other power control schemes: **Eq-TP** with the Equal Transmit Powers of two source nodes and **Eq-RP** with the Equal Received Powers at the relay, i.e. $P_1\Omega_1 = P_2\Omega_2$. The optimal solution of the Eq-TP scheme is $(P_1, P_2) = (\widetilde{P}/2, \widetilde{P}/2)$ and that of the Eq-RP scheme is $(P_1, P_2) = (\frac{\Omega_2}{\Omega_1+\Omega_2}\widetilde{P}, \frac{\Omega_1}{\Omega_1+\Omega_2}\widetilde{P})$.

Fig. 3 shows the BER performance of the three transmit power allocation schemes: **Pro-TP**, **Eq-TP**, and **Eq-RP** for given $\widetilde{P} = 180$ and 360 mW. We can observe that the BER performances of all schemes improve as $\widetilde{P}$ increases. The Pro-TP scheme yields the best BER performance and the performance gap increases as the relay node is close to one of two source nodes. When the relay node is located at the center between two source nodes, the BER is minimized and the BER performance of three schemes are the same. This is because the solution of the three schemes becomes identical to $P_1 = P_2 = \widetilde{P}/2$.

VI. Conclusion

In this letter, we considered the PNC with CCs in a three-node TWRC, where the relay node directly decodes the XORed packet from two source nodes in the MA phase. We proposed novel approximation methods for the BER of the (MA) phase in fast fading channels, which match well with simulation results. Based on them, we also proposed the transmit power allocation strategy for each source node in order to minimize the derived BER expression.

Appendix

The cumulative distribution functions (CDFs) of $X_1 = |h_1\sqrt{P_1}|$ and $X_2 = |h_2\sqrt{P_2}|$ are $F_{X_1}(x_1) = 1 - \exp\left(\frac{-x_1^2}{\Omega_1 P_1}\right)$, $F_{X_2}(x_2) = 1 - \exp\left(\frac{-x_2^2}{\Omega_2 P_2}\right)$, respectively, where $h_1 \sim \mathcal{CN}(0, \Omega_1)$ and $h_2 \sim \mathcal{CN}(0, \Omega_2)$. Since $Z = \min(X_1, X_2)$, the CDF of Z is expressed as $F_Z(z) = 1 - \exp\left(-\frac{z^2}{\Omega_1 P_1}\right)\exp\left(\frac{-z^2}{\Omega_1 P_2}\right) = 1 - \exp\left(-\left(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_1 P_2}\right)z^2\right)$ Therefore, the PDF of Z is given by

$$f_Z(z) = 2\left(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_1 P_2}\right) z \exp\left(-\left(\frac{1}{\Omega_1 P_1} + \frac{1}{\Omega_1 P_2}\right) z^2\right).$$

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